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Length of the Laminar Hypersonic Wake during Ballistic Re-Entry

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1 Introduction

MOST of the studies pertaining to the length of the wake of an object moving at hypersonic speeds in a dense atmosphere refer to steady-state conditions The reason for this assumption is that one can prove that the decelerations involved are very small and contribute very little to the fluid mechanics of the problem For instance, over a period of 10 sec an intercontinental ballistic missile of characteristic length, say of 10 ft, can cover a distance of the order of 100,000 ft With these numbers one can easily see that in the equation of conservation of momentum the order of magnitude of the acceleration $\partial u/\partial t \sim$ (100,000 ft/10 sec)/$\frac{1}{10} \sim 10^3$ ft/sec^2 whereas the order of magnitude of the quantity $u(\partial u/\partial x) \sim$ (100,000 ft/10 sec)^2 1/10 ft $\sim 10^7$ ft/sec^2 For this reason one neglects the nonsteady term in the equation of motion (and similarly in the equations of conservation of mass and energy) and proceeds in solving the steady-state problem As a matter of fact, since for certain calculations one is not interested in the details of the flow very close to the moving object, the assumption that the fluid moves inside the wake with a velocity of the order of 80% of the free-flight velocity is remarkably good

The steady-state results may be interpreted in two ways First, they may be understood in a system of coordinates fixed relative to the body; in this case we go downstream a distance x in order to measure (or calculate) such time independent quantities as electron concentration, wake width, and the like Second, one may define a system of coordinates fixed at the laboratory, form a thin optical slit in front of him, and observe the temporal growth of the wake as the object passes by It is obvious that if the velocity of the object is denoted by U, the coordinate x of the object-fixed system and the coordinate t of the laboratory system will be linked by the simple relation

$$x = Ut \tag{1}$$

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Now assume that the object does not move in a medium of constant mass density Granted that the deceleration which the body will suffer does not influence its fluid mechanical aspects, the wake will develop temporally at each fixed altitude as if it had moved past that point with a constant velocity equal to the one it had when passing through this point In order to find the length of the wake based on a given electron concentration n_e when the object is at point 2, we proceed as follows Assume that the tail of the wake where the electron density becomes n is located at an altitude H_1 From the ballistic diagram of the body (altitude vs velocity vs time for given ballistic coefficient and initial entry conditions) calculate U_1 and the time difference $t_2 - t_1$ Then calculate the length $U_1(t_2 - t_1)$ Knowing these numbers, use the steady-state theory to check if the electron density at the station x corresponding to the altitude H_1 and velocity U_1 is equal to n If not, change the altitude H_1 to another one, and repeat this procedure until it is successful. The length based on this procedure we shall call the "ballistic wake length "

In the next paragraph we shall apply this method assuming thermodynamic equilibrium and a hemisphere-cylinder configuration It goes without saying that this is a highly unrealistic model, but it can well serve as an example illustrating the error that can be made if one assumes that at a given altitude the wake's length is equal to the one that eventually would develop at that altitude if the body had stayed there moving with a constant speed We shall call this last quantity the "constant altitude and velocity wake length "

The problem under discussion has been studied in Ref 1 The present work was undertaken in order to point out that what is called in Ref 1 the nonsteady solution is the steady one, properly understood, and also to show how results can be obtained in a closed form without the need of numerical integrations

2 Calculation

We start with Eq (47) of Ref 2 which is duplicated below for ready reference:

$$\frac{h(x,0)}{RT_0} = \frac{h(0,0)/RT_0}{\left\{1 + \dfrac{[h(0,0)/RT_0]^{1/4}\, x}{0\ 53(\rho_\infty/\rho_0)\, U\, C_D r_0^2}\right\}^{0\ 8}} \tag{2}$$

$h(x,0)$ is the enthalpy at a station at distance x downstream located on the axis of symmetry $h(0,0)$ is the value of the enthalpy at the station $x = 0$ where the gas has expanded to the ambient pressure and from there on can cool only through the mechanism of thermal conduction R is the universal gas content, T_0 and ρ_0 are reference temperature and mass density A discussion of the validity of the forementioned equation can be found in Ref 2 We also recall from Ref 4, that within a good approximation, the following relation is valid:

$$\frac{h(0,0)}{RT_0} = \frac{(\gamma - 1)M^2}{6}\left(\frac{h_\infty}{RT_0}\right) \tag{3}$$

M is the Mach number of the freestream and γ may be taken equal to 1 4 Furthermore, let us assume that we deal with an isothermal atmosphere for which one can write

$$\rho_\infty/\rho_0 = e^{-\alpha H} \tag{4}$$

where α is an appropriate numerical constant ($\alpha = 4\ 15 \times 10^{-5}$ ft^{-1}) and H is the altitude For the density distribution of Eq (4) and given initial conditions of altitude H_0, velocity U_0, and angle of entry θ one can trace diagrams of altitude vs velocity vs time as a function of the ballistic coefficient

$$\beta = W \sin\theta/C_D r_0^2 \tag{5}$$

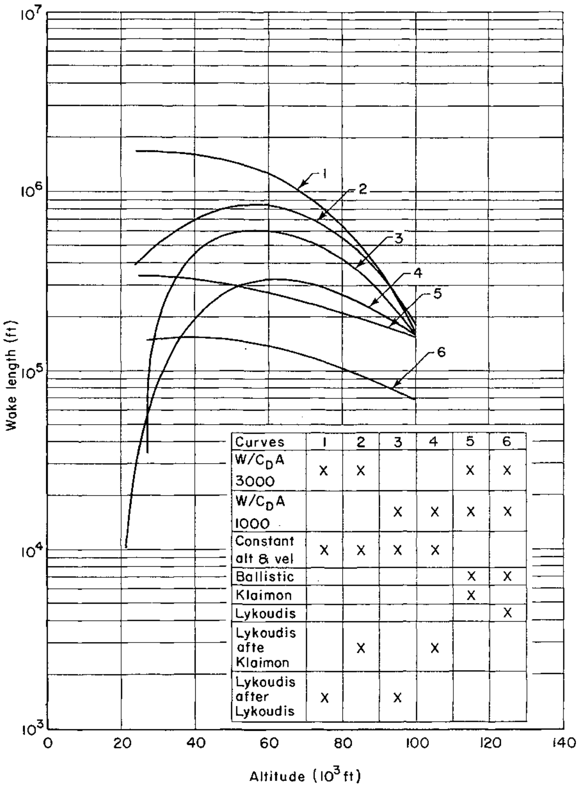
From Ref 5 we quote

$$\frac{U}{U_0} = \exp \frac{-C_D[(\pi/4)r_0^2]}{W \sin\theta} \frac{\rho_0}{2\alpha} e^{-\alpha H} \tag{6}$$

and

$$t_2 - t_1 = \frac{1}{\alpha U_0 \sin\theta} \int_{H_1}^{H_2} \left(\frac{1}{e^{-\alpha H}}\right) \exp \frac{C_D[(\pi/4)r_0^2]}{W \sin\theta} \frac{\rho_0}{2\alpha} e^{-\alpha H} \, dH \tag{7}$$

Following the procedure outlined in the introduction, the "ballistic length" of the laminar wake of a hemisphere-cylinder of radius r_0 at a given altitude H_2 will be calculated from Eq (2) by making use of the following substitutions: $h(x,0)/RT_0$ will be equal to the value it has at the altitude H_1 so that the electron concentration will be n_e The density ratio ρ_∞/ρ_0 will be evaluated at the same point The quantity $[h(0,0)/RT_0]$ may be evaluated either by using direct gas tables or by making use of the formula provided by Eq (3); the velocity will be the one yielded by Eq (6) for the altitude H_1 The quantity x will be made equal to $U_1(t_2 - t_1)$ and the time interval $(t_2 - t_1)$ will be computed from Eq (7) †



Curves	1	2	3	4	5	6
W/C_DA 3000	x	x			x	x
W/C_DA 1000			x	x	x	x
Constant alt & vel	x	x	x	x		
Ballistic					x	x
Klaimon					x	
Lykoudis						x
Lykoudis afte Klaimon		x		x		
Lykoudis after Lykoudis	x		x			

Fig 1 Wake length based on $n_e = 10^7$ electrons/cm^3 and a hemisphere cylinder configuration 30 cm in radius

From the foregoing it becomes evident that Eq (2) contains H_1 as the only unknown, however no formal solution is possible; on the other hand a few guesses are sufficient before Eq (2) can be satisfied

After the evaluation of H_1, the "ballistic length" of the trail, based on the electron concentration n_e, is given by the simple relation $L_1 = (H_1 - H_2)/\sin\theta$

† Equations (6) and (7) can be substituted by ballistic data such as those published in Ref 6

3 Examples and Conclusions

Figure 1 shows the numerical results obtained by using the method previously described for the calculation of the "ballistic length" of the wake for the case of two ballistic coefficients, namely, $W/C_DA = 1000$ and 3000 The initial conditions taken for the re-entry are at an angle with the horizontal of 20° and a re-entry velocity of 23,900 fps starting the trajectory at an altitude of 400,000 ft This is the case studied in detail in Ref 6 For both ballistic coefficients it was found that the length remains remarkably the same, as already found in Ref 1 If one compares this result with the "constant altitude and velocity length," then the dependence of the ballistic coefficient is very strong for the latter This behavior can be explained as follows Given an altitude, the trajectory with the smaller ballistic coefficient will have a smaller velocity and hence will have a smaller amount of kinetic energy to dissipate From Eq (2) it can be seen that the "constant altitude and velocity length" will depend directly and solely on this energy On the other hand, the "ballistic length" corresponding to a given altitude depends on the history traced downstream during the re-entry, and hence the ballistic coefficient becomes less significant if the tail of the wake is located at an altitude where the trajectories are not yet strongly dependent on the ballistic coefficient As a matter of fact, in this last case a quick estimate of the "ballistic length" may be obtained by solving Eq (2) for the density ratio ρ_∞/ρ_0 using a rough guess for the time interval $(t_2 - t_1)$ and the initial re-entry velocity

Figure 1 shows also the numerical data of Ref 1 The trend for all the cases studied in common are the same, but the numerical values seem to be apart by a factor of two

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Production of Intense Radiation Heat Pulses

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At the re-entry velocities typical of interplanetary flights, it is likely that very high heat transfer rates to the entering vehicle will be encountered due to radiation from the hot gases surrounding the body[1 2] These rates can approach 50 kw/cm^2 in some cases In order to study the behavior of protective materials for this environment, it is desirable that steady-state simulation of the conditions be provided in the

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